

UK INTERMEDIATE MATHEMATICAL CHALLENGE

THURSDAY 7TH FEBRUARY 2008

Organised by the **United Kingdom Mathematics Trust**
and supported by



The Actuarial Profession
making financial sense of the future

RULES AND GUIDELINES (to be read before starting)

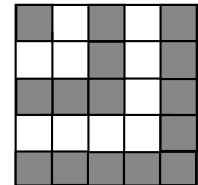
1. Do not open the paper until the Invigilator tells you to do so.
2. Time allowed: **1 hour**.
No answers, or personal details, may be entered after the allowed hour is over.
3. The use of rough paper is allowed; **calculators** and measuring instruments are **forbidden**.
4. Candidates in England and Wales must be in School Year 11 or below.
Candidates in Scotland must be in S4 or below.
Candidates in Northern Ireland must be in School Year 12 or below.
5. **Use B or HB pencil only**. Mark *at most one* of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
6. *Do not expect to finish the whole paper in 1 hour*. Concentrate first on Questions 1-15. When you have checked your answers to these, have a go at some of the later questions.
7. Five marks are awarded for each correct answer to Questions 1-15.
Six marks are awarded for each correct answer to Questions 16-25.
Each incorrect answer to Questions 16-20 loses 1 mark.
Each incorrect answer to Questions 21-25 loses 2 marks.
8. Your Answer Sheet will be read only by a *dumb machine*. **Do not write or doodle on the sheet except to mark your chosen options**. The machine 'sees' all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of rubber stuck to the page, the machine will 'see' a mark and interpret this mark in its own way.
9. The questions on this paper challenge you to **think**, not to guess. You get more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. The UK IMC is about solving interesting problems, not about lucky guessing.

The UKMT is a registered charity

<http://www.ukmt.org.uk>

- How many hours are there in this week?
A 24 B 70 C 84 D 148 E 168
- Which is the largest prime number that divides exactly into the number equal to $2 + 3 + 5 \times 7$?
A 2 B 3 C 5 D 7 E 11
- What is the value of $0.75 \div \frac{3}{4}$?
A 0.5 B 1 C 1.5 D 2 E 2.5

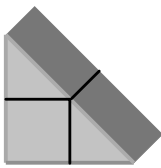
- What percentage of the large 5×5 square is shaded?
A 40% B 60% C $66\frac{2}{3}\%$ D 75% E 80%



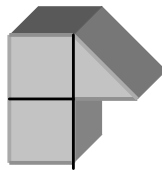
- Which of the following is *not* equal to a whole number?

A $\frac{594}{5 + 9 + 4}$ B $\frac{684}{6 + 8 + 4}$ C $\frac{756}{7 + 5 + 6}$ D $\frac{873}{8 + 7 + 3}$ E $\frac{972}{9 + 7 + 2}$

- Four of these shapes can be placed together to make a cube. Which is the odd one out?



A



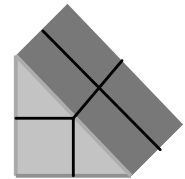
B



C



D



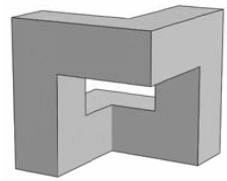
E

- The square of a non-zero number is equal to 70% of the original number. What is the original number?
A 700 B 70 C 7 D 0.7 E 0.07
- In a certain year, there were exactly four Tuesdays and exactly four Fridays in October. On what day of the week did Halloween, October 31st, fall that year?
A Monday B Wednesday C Thursday D Saturday E Sunday
- A solid wooden cube is painted blue on the outside. The cube is then cut into 27 smaller cubes of equal size. What fraction of the total surface area of these new cubes is blue?
A $\frac{1}{6}$ B $\frac{1}{5}$ C $\frac{1}{4}$ D $\frac{1}{3}$ E $\frac{1}{2}$
- Two sides of a triangle have lengths 6 cm and 5 cm. Perry suggests the following possible values for the perimeter of the triangle: (i) 11 cm (ii) 15 cm (iii) 24 cm. Which of Perry's suggestions could be correct?
A (i) only B (i) or (ii) C (ii) only D (ii) or (iii) E (iii) only

11. S is 25% of 60 60 is 80% of U 80 is M % of 25

What is $S + U + M$?

- A 100 B 103 C 165 D 330 E 410
12. The sculpture 'Cubo Vazado' [Emptied Cube] by the Brazilian artist Franz Weissmann is formed by removing cubical blocks from a solid cube to leave the symmetrical shape shown. If all the edges have length 1, 2 or 3 units, what is the surface area of the sculpture in square units?

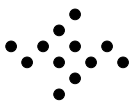


- A 36 B 42 C 48 D 54 E 60
13. The mean of a sequence of 64 numbers is 64. The mean of the first 36 numbers is 36. What is the mean of the last 28 numbers?
- A 28 B 44 C 72 D 100 E 108

14. Sam is holding two lengths of rope by their mid-points. Pat chooses two of the loose ends at random and ties them together. What is the probability that Sam now holds one untied length of rope and one tied loop of rope?

- A $\frac{1}{2}$ B $\frac{1}{3}$ C $\frac{1}{4}$ D $\frac{1}{5}$ E $\frac{1}{6}$

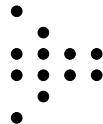
15. A designer wishes to use two copies of the logo shown on the right to create a pattern, without any of the dots overlapping. Which one of the following could be made?



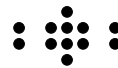
A



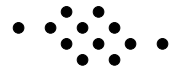
B



C



D



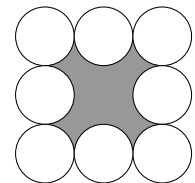
E

16. The first two terms of a sequence are $\frac{2}{3}$ and $\frac{4}{5}$. Each term after the second term is the average (mean) of the two previous terms. What is the fifth term in the sequence?

- A $\frac{5}{34}$ B $\frac{1}{2}$ C $\frac{10}{13}$ D $\frac{3}{4}$ E $\frac{10}{11}$

17. The shaded region is bounded by eight equal circles with centres at the corners and midpoints of the sides of a square.

The perimeter of the square has length 8. What is length of the perimeter of the shaded region?



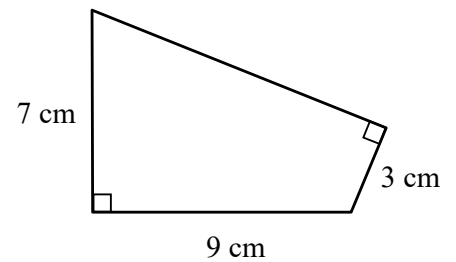
- A π B 2π C 8 D 3π E 4π

18. In the calculation $1003 \div 4995 = 0.2\dot{0}0\dot{8}$, the number $0.2\dot{0}0\dot{8}$ represents the recurring decimal fraction $0.2008008008008\dots$. When the answers to the following calculations are arranged in numerical order, which one is in the middle?

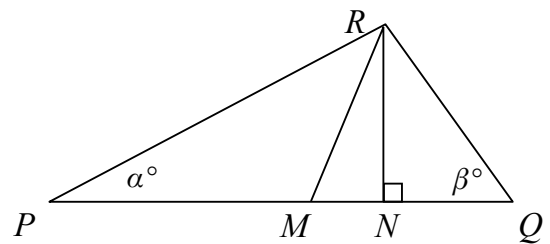
- A $226 \div 1125 = 0.200\dot{8}$ B $251 \div 1250 = 0.2008$ C $497 \div 2475 = 0.200\dot{8}$
D $1003 \div 4995 = 0.2\dot{0}0\dot{8}$ E $2008 \div 9999 = 0.2\dot{0}0\dot{8}$

19. Which of the following is equal to $(1 + x + y)^2 - (1 - x - y)^2$ for all values of x and y ?
- A $4x$ B $2(x^2 + y^2)$ C 0 D $4xy$ E $4(x + y)$

20. What, in cm^2 , is the area of this quadrilateral?
- A 48 B 50 C 52 D 54 E 56



21. In triangle PQR , $\angle QPR = \alpha^\circ$ and $\angle PQR = \beta^\circ$, where $\alpha < \beta$. The line RM bisects $\angle PRQ$ and RN is the perpendicular from R to the line PQ . What is the size, in degrees, of $\angle MRN$?

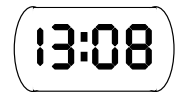
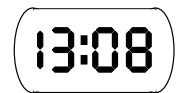


- A $\frac{180 - (\alpha + \beta)}{2}$ B $\frac{\beta - \alpha}{2}$ C $\frac{\alpha + 2\beta}{2}$ D $\frac{360 - \alpha - 2\beta}{2}$ E $\frac{\alpha + \beta}{2}$

22. At a cinema, a child's ticket costs £4.20 and an adult's ticket costs £7.70. When a group of adults and children went to see a film, the total cost was £ C . Which of the following is a possible value of C ?

- A 91 B 92 C 93 D 94 E 95

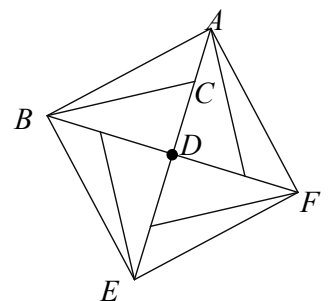
23. Beatrix has a 24-hour digital clock on a glass table-top next to her desk. When she looked at the clock at 13:08, she noticed that the reflected display also read 13:08, as shown.



How many times in a 24-hour period do the display and its reflection give the same time?

- A 12 B 36 C 48 D 72 E 96

24. The diagram has order 4 rotational symmetry about D . If angle ABC is 15° and the area of $ABEF$ is 24 cm^2 , what, in cm , is the length of CD ?



- A 1 B $\sqrt{3}$ C 2 D $\sqrt{5}$ E $2\sqrt{3} - 1$

25. A garden has the shape of a right-angled triangle with sides of length 30, 40 and 50. A straight fence goes from the corner with the right-angle to a point on the opposite side, dividing the garden into two sections which have the same perimeter. How long is the fence?

- A 25 B $8\sqrt{3}$ C $5\sqrt{11}$ D $5\sqrt{39}$ E $12\sqrt{5}$